

The Quark Mass Problem and CP -Violation

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Abstract

A simple breaking of the subnuclear democracy among the quarks leads to a mixing between the second and the third family, in agreement with observation. Introducing the mixing between the first and the second family, one finds an interesting pattern of maximal CP -violation as well as a complete determination of the elements of the CKM matrix and of the unitarity triangles.

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In the standard electroweak model both the masses of the quarks as well as the weak mixing angles appear as free parameters. Further insights into the yet unknown dynamics of mass generation would imply steps beyond the physics of the electroweak standard model. At present it seems far too early to attempt an actual solution of the dynamics of mass generation, and one is invited to follow a strategy similar to the one which led eventually to the solution of the strong interaction dynamics by QCD, by looking for specific patterns and symmetries as well as specific symmetry violations.

It is well-known that the mass spectra of the quarks are dominated essentially by the masses of the members of the third family, i. e. by t and b . A clear hierarchical pattern exists. Furthermore the masses of the first family are small compared to those of the second one. Moreover, the CKM-mixing matrix exhibits a hierarchical pattern – the transitions between the second and third family as well as between the first and the third family are small compared to those between the first and the second family.

It was emphasized years ago¹⁾ that the observed hierarchies indicate that nature seems to be close to the so-called “rank-one” limit, in which all mixing angles vanish and both the u- and d-type mass matrices are proportional to the rank-one matrix

$$M_0 = \text{const.} \cdot \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (1)$$

Whether the dynamics of the mass generation allows that this limit can be achieved in a consistent way remains an unsolved issue, depending on the dynamical details of mass generation. Encouraged by the observed hierarchical pattern of the masses and the mixing parameters, we shall assume that this is the case. In itself it is a non-trivial constraint and can be derived from imposing a chiral symmetry, as emphasized in ref. (2). This symmetry ensures that an electroweak doublet which is massless remains unmixed and is coupled to the W -boson with full strength.

As soon as the mass is introduced, at least for one member of the doublet, the symmetry is violated and mixing phenomena are expected to show up. That way a chiral evolution of the CKM matrix can be constructed.²⁾ At the first stage only the t and b quark masses are introduced, due to their non-vanishing coupling to the scalar “Higgs” field. The CKM-matrix

is unity in this limit. At the next stage the second generation acquires a mass. Since the (u, d) -doublet is still massless, only the second and the third generations mix, and the CKM-matrix is given by a real 2×2 rotation matrix in the $(c, s) - (t, b)$ subsystem, describing e. g. the mixing between s and b . Only at the next step, at which the u and d masses are introduced, does the full CKM-matrix appear, described in general by three angles and one phase. Only at this step CP -violation can occur. Thus it is the generation of mass for the first family which is responsible for the violation of the CP -symmetry.

It has been emphasized some time ago^{3,4)} that the rank-one mass matrix (see eq. (1)) can be expressed in terms of a “democratic mass matrix”:

$$M_0 = c \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (2)$$

which exhibits an $S(3)_L \times S(3)_R$ symmetry. Writing down the mass eigenstates in terms of the eigenstates of the “democratic” symmetry, one finds e.g. for the u -quark channel:

$$\begin{aligned} u^0 &= \frac{1}{\sqrt{2}}(u_1 - u_2) \\ c^0 &= \frac{1}{\sqrt{6}}(u_1 + u_2 - 2u_3) \\ t^0 &= \frac{1}{\sqrt{3}}(u_1 + u_2 + u_3). \end{aligned} \quad (3)$$

Here u_1, \dots are the symmetry eigenstates. Note that u^0 and c^0 are massless in the limit considered here, and any linear combination of the first two state vectors given in eq. (3) would fulfill the same purpose, i. e. the decomposition is not unique, only the wave function of the coherent state t^0 is uniquely defined. This ambiguity will disappear as soon as the symmetry is violated.

The wave functions given in eq. (3) are reminiscent of the wave functions of the neutral pseudoscalar mesons in QCD in the $SU(3)_L \times SU(3)_R$ limit:

$$\begin{aligned} \pi_0^0 &= \frac{1}{\sqrt{2}}(\bar{u}u - \bar{d}d) \\ \eta_0 &= \frac{1}{\sqrt{6}}(\bar{u}u + \bar{d}d - 2\bar{s}s) \\ \eta_0' &= \frac{1}{\sqrt{3}}(\bar{u}u + \bar{d}d + \bar{s}s). \end{aligned} \quad (4)$$

(Here the lower index denotes that we are considering the chiral limit). Also the mass spectrum of these mesons is identical to the mass spectrum of the quarks in the “democratic” limit: two mesons (π_0^0, η_0) are massless and act as Nambu–Goldstone bosons, while the third coherent state η'_0 is not massless due to the QCD anomaly.

In the chiral limit the (mass)²–matrix of the neutral pseudoscalar mesons is also a “democratic” mass matrix when written in terms of the $(\bar{q}q)$ –eigenstates $(\bar{u}u)$, $(\bar{d}d)$ and $(\bar{s}s)$ ⁵⁾:

$$M^2(ps) = \lambda \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}, \quad (5)$$

where the strength parameter λ is given by $\lambda = M^2(\eta'_0) / 3$. The mass matrix (5) describes the result of the QCD–anomaly which causes strong transitions between the quark eigenstates (due to gluonic annihilation effects enhanced by topological effects). Likewise one may argue that analogous transitions are the reason for the lepton–quark mass hierarchy. Here we shall not speculate about a detailed mechanism of this type, but merely study the effect of symmetry breaking.

In the case of the pseudoscalar mesons the breaking of the symmetry down to $SU(2)_L \times SU(2)_R$ is provided by a direct mass term $m_s \bar{s}s$ for the s–quark. This implies a modification of the (3,3) matrix element in eq. (5), where λ is replaced by $\lambda + M^2(\bar{s}s)$ where $M^2(\bar{s}s)$ is given by $2M_K^2$, which is proportional to $\langle \bar{s}s \rangle_0$, the expectation value of $\bar{s}s$ in the QCD vacuum. This direct mass term causes the violation of the symmetry and generates at the same time a mixing between η_0 and η'_0 , a mass for the η_0 , and a mass shift for the η'_0 .

It would be interesting to see whether an analogue of the simplest violation of this kind of symmetry violation of the “democratic” symmetry which describes successfully the mass and mixing pattern of the $\eta - \eta'$ –system is also able to describe the observed mixing and mass pattern of the second and third family of leptons and quarks. This was discussed recently⁶⁾. Let us replace the (3,3) matrix element in eq. (2) by $1 + \varepsilon_i$; ($i = u$ (u–quarks), d (d–quarks) respectively). The small real parameters ε_i describe the departure from democratic symmetry and lead

- a) to a generation of mass for the second family and
- b) to a flavour mixing between the third and the second family. Since ε is directly related

(see below) to a fermion mass and the latter is not restricted to be positive, ε can be positive or negative. (Note that a negative Fermi–Dirac mass can always be turned into a positive one by a suitable γ_5 –transformation of the spin $\frac{1}{2}$ field). Since the original mass term is represented by a symmetric matrix, we take ε to be real.

It is instructive to rewrite the mass matrix in the hierarchical basis, where one obtains in the case of the down–type quarks:

$$M = c_l \begin{pmatrix} 0 & 0 & 0 \\ 0 & +\frac{2}{3}\varepsilon_u & -\frac{\sqrt{2}}{3}\varepsilon_u \\ 0 & -\frac{\sqrt{2}}{3}\varepsilon_u & 3 + \frac{1}{3}\varepsilon_u \end{pmatrix}. \quad (6)$$

In lowest order of ε one finds the mass eigenvalues $m_s = \frac{2}{9}\varepsilon_d \cdot m_b, m_b = m_{b^0}, \Theta_{s,b} = |\sqrt{2} \cdot \varepsilon_d/9|$.

The exact mass eigenvalues and the mixing angle are given by:

$$\begin{aligned} m_1/c_d &= \frac{3 + \varepsilon_d}{2} - \frac{3}{2} \sqrt{1 - \frac{2}{9}\varepsilon_d + \frac{1}{9}\varepsilon_d^2} \\ m_2/c_d &= \frac{3 + \varepsilon_d}{2} + \frac{3}{2} \sqrt{1 - \frac{2}{9}\varepsilon_d + \frac{1}{9}\varepsilon_d^2} \\ \sin \Theta_{(s,b)} &= \frac{1}{\sqrt{2}} \left(1 - \frac{1 - \frac{1}{9}\varepsilon_d}{(1 - \frac{2}{9}\varepsilon_d + \frac{1}{9}\varepsilon_d^2)^{1/2}} \right)^{1/2}. \end{aligned} \quad (7)$$

The ratio m_s/m_b is allowed to vary in the range $0.022 \dots 0.044$ (see ref. (7)). According to eq. (7) one finds ε_d to vary from $\varepsilon_d = 0.11$ to 0.21 . The associated s – b mixing angle varies from $\Theta(s, b) = 1.0^\circ$ ($\sin \Theta = 0.018$) and $\Theta(s, b) = 1.95^\circ$ ($\sin \Theta = 0.034$). As an illustrative example we use the values $m_b(1\text{GeV}) = 5200 \text{ MeV}$, $m_s(1\text{GeV}) = 220 \text{ MeV}$. One obtains $\varepsilon_d = 0.20$ and $\sin \Theta(s, b) = 0.032$.

To determine the amount of mixing in the (c, t) –channel, a knowledge of the ratio m_c/m_t is required. As an illustrative example we take $m_c(m_t)/m_t(m_t) = 0.005$, which corresponds to $m_t(m_t) \cong 170 \text{ GeV}$, $m_c(1\text{GeV}) \cong 1.35 \text{ GeV}$. In this case one finds $\varepsilon_u = 0.023$ and $\Theta(c, t) = 0.21^\circ$ ($\sin \Theta(c, t) = 0.004$).

The actual weak mixing between the third and the second quark family is a combined effect of the two family mixings described above. The symmetry breaking given by the ε –parameter can be interpreted, as done in eq. (7), as a direct mass term for the u_3, d_3 fermion. However,

a direct fermion mass term need not be positive, since its sign can always be changed by a suitable γ_5 -transformation. What counts for our analysis is the relative sign of the m_s -mass term in comparison to the m_c -term, discussed previously. Thus two possibilities must be considered:

- a) Both the m_s - and the m_c -term have the same relative sign with respect to each other, i. e. both ε_d and ε_u are positive, and the mixing angle between the second and third family is given by the difference $\Theta(sb) - \Theta(ct)$. This possibility seems to be ruled out by experiment, since it would lead to $V_{cb} < 0.03$.
- b) The relative signs of the breaking terms ε_d and ε_u are different, and the mixing angle between the (s, b) and (c, t) systems is given by the sum $\Theta(sb) + \Theta(ct)$. Thus we obtain $V_{cb} \cong \sin(\Theta(sb) + \Theta(ct))$.

According to the range of values for m_s discussed above, one finds $V_{cb} \cong 0.022 \dots 0.038$. For example, for $m_s(1\text{GeV}) = 220\text{ MeV}$, $m_c(1\text{GeV}) = 1.35\text{ GeV}$, one finds $V_{cb} \cong 0.036$.

The experiments give $V_{cb} = 0.032 \dots 0.048^8)$. We conclude from the analysis given above that our ansatz for the symmetry breaking reproduces the lower part of the experimental range. According to a recent analysis the experimental data are reproduced best for $V_{cb} = 0.038 \pm 0.003^9)$. We obtain consistency with experiment only if the ratio m_s/m_b is relatively large implying $m_s(1\text{GeV}) \geq 180\text{ MeV}$. Note that recent estimates of m_s (1GeV) give values in the range $180 \dots 200\text{ MeV}^{10)}$.

It is remarkable that the simplest ansatz for the breaking of the “democratic symmetry”, one which nature follows in the case of the pseudoscalar mesons, is able to reproduce the experimental data on the mixing between the second and third family. We interpret this as a hint that the eigenstates of the symmetry, not the mass eigenstates, play a special rôle in the physics of flavour, a rôle which needs to be investigated further.

The next step is to introduce the mass of the d quark, but keeping m_u massless. We regard this sequence of steps as useful due to the fact that the mass ratios m_u/m_c and m_u/m_t are about one order of magnitude smaller than the ratios m_d/m_s and m_d/m_b respectively. It is well-known that the observed magnitude of the mixing between the first and the second family can be reproduced well by a specific texture of the mass matrix^{11),12)}. We shall

incorporate this here and take the following structure for the mass matrix of the down-type quarks:

$$M_d = \begin{pmatrix} 0 & D_d & 0 \\ D_d^* & C_d & B_d \\ 0 & B_d & A_d \end{pmatrix}. \quad (8)$$

Here $A_d = c_d(3 + \frac{1}{3}\varepsilon_d)$, $B_d = -\sqrt{2}/3 \cdot \varepsilon_d \cdot c_d$, $C_d = \frac{2}{3} \cdot \varepsilon_d \cdot c_d$. At this stage the mass matrix of the up-type quarks remains in the form (6). The CKM matrix elements V_{us} , V_{cd} and the ratios V_{ub}/V_{cb} , V_{td}/V_{ts} can be calculated in this limit. One finds in lowest order:

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}}, \quad V_{cd} \approx \sqrt{\frac{m_d}{m_s}}, \quad \frac{V_{ub}}{V_{cb}} \approx 0, \quad \frac{V_{td}}{V_{ts}} \approx \sqrt{\frac{m_d}{m_s}}. \quad (9)$$

An interesting implication of the ansatz (8) is the vanishing of CP violation. Although the mass matrix (5) contains a complex parameter D_d , its phase can be rotated away due to the fact that m_u is still massless, and a phase rotation of the u -field does not lead to any observable consequences. The vanishing of CP violation can be seen as follows. Considering two hermitian mass matrices M_u and M_d in general, one may define a commutator like

$$[M_u, M_d] = i\mathcal{C} \quad (10)$$

The final step is to introduce the mass of the u quark. The mass matrix M_u takes the form:

$$M_u = \begin{pmatrix} 0 & D_u & 0 \\ D_u^* & C_u & B_u \\ 0 & B_u & A_u \end{pmatrix}. \quad (11)$$

(Here A_u etc. are defined analogously as in e.g. (8)). Once the mixing term $D_u = |D_u|e^{i\sigma}$ for the u -quark is introduced, CP violation appears. For the determinant of the commutator (6) we find:

$$\text{Det } \mathcal{C} \cong T \sin \sigma, \quad (12)$$

$$\begin{aligned} T = & 2|D_u D_d| [(A_u B_d - B_u A_d)^2 - |D_u|^2 B_d^2 - B_u^2 |D_d|^2 \\ & - (A_u B_d - B_u A_d)(C_u B_d - B_u C_d)] . \end{aligned} \quad (13)$$

The phase σ determines the strength of CP violation. The diagonalization of the mass matrices M_d and M_u leads to the eigenvalues m_i ($i = u, d, \dots$). Note that m_u and m_d appear to be negative. By a suitable γ_5 -transformation of the quark fields one can arrange them to

be positive. Collecting the lowest order terms in the CKM matrix, one obtains:

$$V_{us} \approx \sqrt{\frac{m_d}{m_s}} - \sqrt{\frac{m_u}{m_c}} e^{i\sigma}, \quad V_{cd} \approx \sqrt{\frac{m_u}{m_c}} - \sqrt{\frac{m_d}{m_s}} e^{i\sigma} \quad (14)$$

and

$$\frac{V_{ub}}{V_{cb}} \approx -\sqrt{\frac{m_u}{m_c}}, \quad \frac{V_{td}}{V_{ts}} \approx -\sqrt{\frac{m_d}{m_s}}. \quad (15)$$

The relations for V_{us} and V_{cd} were obtained previously¹²⁾. However then it was not noted that the relative phase between the two ratios might be relevant for CP violation. A related discussion can be found in ref. [15].

According to eq. (12) the strength of CP violation depends on the phase σ . If we keep the modulus of the parameter D_u constant, but vary the phase from zero to 90° , the strength of CP violation varies from zero to a maximal value given by eq. (12), which is obtained for $\sigma = 90^\circ$. We conclude that CP violation is maximal for $\sigma = 90^\circ$. In this case the element D_u would be purely imaginary, if we set the phase of the matrix element D_d to be zero. As discussed above, this can always be arranged.

In our approach the CP -violating phase also enters in the expressions for V_{us} and V_{cd} (Cabibbo angle). As discussed already in ref. [12], the Cabibbo angle is fixed by the difference of $\sqrt{m_d/m_s}$ and $\sqrt{m_u/m_c} \times \text{phase factor}$. The second term contributes a small correction (of order 0.06) to the leading term, which according to the mass ratios given in ref. [8] is allowed to vary between 0.20 and 0.24. For our subsequent discussion we shall use $0.218 \leq |V_{us}| \leq 0.224$ [8]. If the phase parameter multiplying $\sqrt{m_u/m_c}$ were zero or $\pm 180^\circ$ (i.e. either the difference or sum of the two real terms would enter), the observed magnitude of the Cabibbo angle could not be reproduced. Thus a phase is needed, and we find within our approach purely on phenomenological grounds that CP violation must be present if we request consistency between observation and our result (14).

An excellent description of the magnitude of V_{us} is obtained for a phase angle of 90° . In this case one finds:

$$|V_{us}|^2 \approx \left(1 - \frac{m_d}{m_s}\right) \left(\frac{m_d}{m_s} + \frac{m_u}{m_c}\right), \quad (16)$$

where approximations are made for V_{us} to a better degree of accuracy than that in eq. (14).

Using $|V_{us}| = 0.218 \dots 0.224$ and $m_u/m_c = 0.0028 \dots 0.0048$ we obtain $m_d/m_s \approx 0.045 \dots 0.05$.

This corresponds to $m_s/m_d \approx 20...22$, which is entirely consistent with the determination of m_s/m_d , based on chiral perturbation theory [7]: $m_s/m_d = 17...25$. This example shows that the phase angle must be in the vicinity of 90^0 . Fixing m_u/m_c to its central value and varying m_d/m_s throughout the allowed range, we find $\sigma \approx 66^0...110^0$.

The case $\sigma = 90^0$, favoured by our analysis, deserves a special attention. It implies that in the sequence of steps discussed above the term D_u generating the mass of the u -quark is purely imaginary, and hence CP violation is maximal. It is of high interest to observe that nature seems to prefer this case. A purely imaginary term D_u implies that the algebraic structure of the quark mass matrix is particularly simple. Its consequences need to be investigated further and might lead the way to an underlying internal symmetry responsible for the pattern of masses.

Finally we explore the consequences of our approach to the unitarity triangle, i. e., the triangle formed by the CKM matrix elements V_{ub}^* , V_{td} and $s_{12}V_{cb}$ ($s_{12} = \sin \theta_{12}$, θ_{12} : Cabibbo angle) in the complex plane (we shall use the definitions of the angles α , β and γ as given in ref. [8]). For $\sigma = 90^0$ we obtain:

$$\alpha \approx 90^0, \quad \beta \approx \arctan \sqrt{\frac{m_u}{m_c} \cdot \frac{m_s}{m_d}}, \quad \gamma \approx 90^0 - \beta. \quad (17)$$

Thus the unitarity triangle is a rectangular triangle. We note that the unitarity triangle and the triangle formed in the complex phase by V_{us} , $\sqrt{m_d/m_s}$ and $\sqrt{m_u/m_c}$ are similar rectangular triangles, related by a scale transformation. Using as input $m_u/m_c = 0.0028...0.0048$ and $m_s/m_d = 20...22$ as discussed above, we find $\beta \approx 13^0...18^0$, $\gamma \approx 72^0...76^0$, and $\sin 2\beta \approx \sin 2\gamma \approx 0.45...0.59$. These values are consistent with the experimental constraints [16].

We have shown that a simple pattern for the generation of masses for the first family of leptons and quarks leads to an interesting and predictive pattern for the violation of CP symmetry. The observed magnitude of the Cabibbo angle requires CP violation to be maximal or at least near to its maximal strength. The ratio V_{ub}/V_{cb} as well as V_{td}/V_{ts} are given by $\sqrt{m_u/m_c}$ and $\sqrt{m_d/m_s}$ respectively. In the case of maximal CP violation the unitarity triangle is rectangular ($\alpha = 90^0$), the angle β can vary in the range $13^0...18^0$ ($\sin 2\beta = \sin 2\gamma \approx 0.45...0.59$). It remains to be seen whether the future experiments, e.g. the measurements

of the CP asymmetry in the B decays, confirm these values.

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